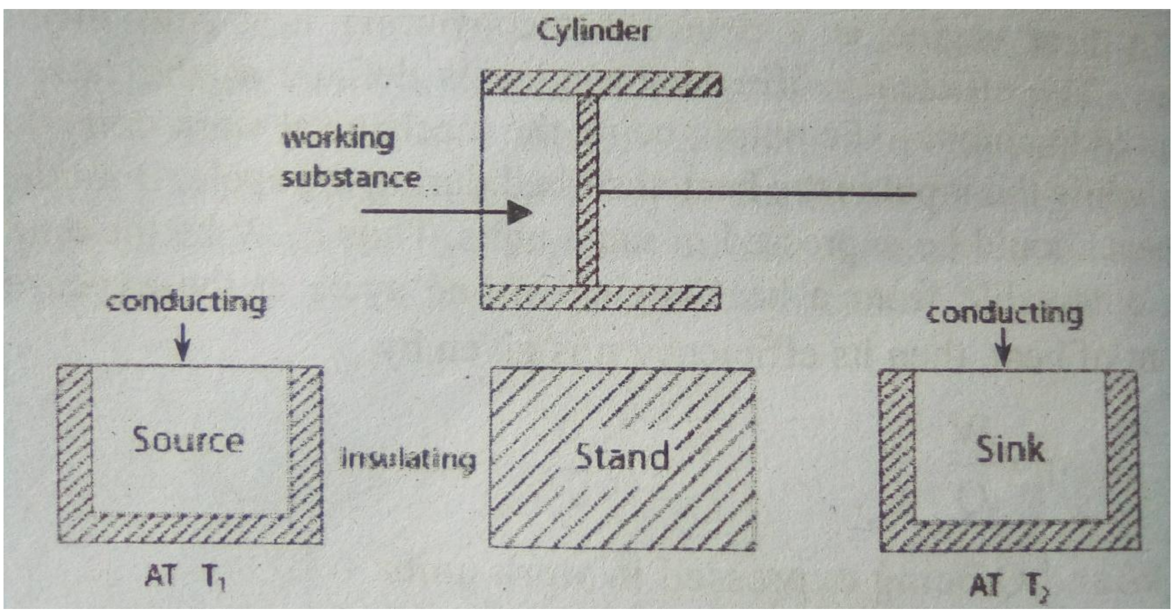
Carnot cycle

The Carnot cycle: A cycle in which the working substance starting from a given condition of temperature, pressure and volume is made to undergo two successive expressions (one isothermal and another adiabatic), and then two successive compressions (one isothermal and another adiabatic) at the end of which the working substance is brought back to its initial condition, is called Carnot's cycle.

i) **a cylinder:** the walls of the cylinder are perfectly non-conducting but its bottom is perfectly conducting. The cylinder is provided with a non-conducting piston P which moves horizontally inside the cylinder without friction. The working substance inside the cylinder is a perfect gas.

ii) **a source:** it is a hot body or heat reservoir of unlimited heat capacity, kept always at a constant high temperature T_1K. The temperature of the body is supposed to remain constant even if an amount of heat is taken from it.



iii) **a sink:** it is a cold body of unlimited heat-receiving capacity, kept at a constant temperature T_2K (lower than T_1). Its temperature is supposed to remain constant even if an amount of heat is given to it.

iv) **a non-conducting stand:** it is a block of non-conducting material.

Carnot's theorem: Carnot's theorem, developed in 1824 by Nicolas Léonard Sadi Carnot, also called **Carnot's rule**, is a principle that specifies limits on the maximum efficiency any heat engine can obtain. The efficiency of a Carnot engine depends solely on the temperatures of the hot and cold reservoirs.

Carnot's theorem states:

- All heat engines between two heat reservoirs are less efficient than a Carnot heat engine operating between the same reservoirs.
- Every Carnot heat engine between a pair of heat reservoirs is equally efficient, regardless of the working substance employed or the operation details.

The formula for this maximum efficiency is

$$\eta_{max} = \frac{W}{Q_H} = 1 - \frac{T_C}{T_H}$$

Where, η_{max} is the maximum efficiency, W is the work down by the system (energy exiting the system as work), Q_H is the heat put into the system (heat energy entering the system), T_C is the absolute temperature of the cold reservoir, T_H is the absolute temperature of the hot reservoir, and the efficiency η is the ratio of the work done by the engine to the heat drawn out of the hot reservoir.

The second law of thermodynamics puts a fundamental limit on the thermal efficiency of all heat engines. Even an ideal, frictionless engine can't convert anywhere near 100% of its input heat into work. The limiting factors are the temperature at which the heat enters the engine, T_H, and the temperature of the environment into which the engine exhausts its waste heat, T_c , measured in an absolute scale, such as the Kelvin or Rankine scale.

The **Carnot cycle** when acting as a heat engine consists of the following steps:

1. **Reversible isothermal expansion of the gas at the "hot" temperature, *T_H* (isothermal heat addition or absorption).** During this step (1 to 2 on Figure 1, *A* to *B* in Figure 2) the gas is allowed to expand and it does work on the surroundings. The temperature of the gas does not change during the process, and thus the expansion is isothermic. The gas expansion is propelled by absorption of heat energy Q_1 and of entropy $\Delta S_H = \frac{Q_H}{T_H}$ from the high temperature reservoir.
2. **Isentropic (reversible adiabatic) expansion of the gas (isentropic work output).** For this step (2 to 3 on Figure 1, *B* to *C* in Figure 2) the piston and cylinder are assumed to be thermally insulated, thus they neither gain nor lose heat. The gas continues to expand, doing work on the surroundings, and losing an equivalent amount of internal energy. The gas expansion causes it to cool to the "cold" temperature, T_C. The entropy remains unchanged.

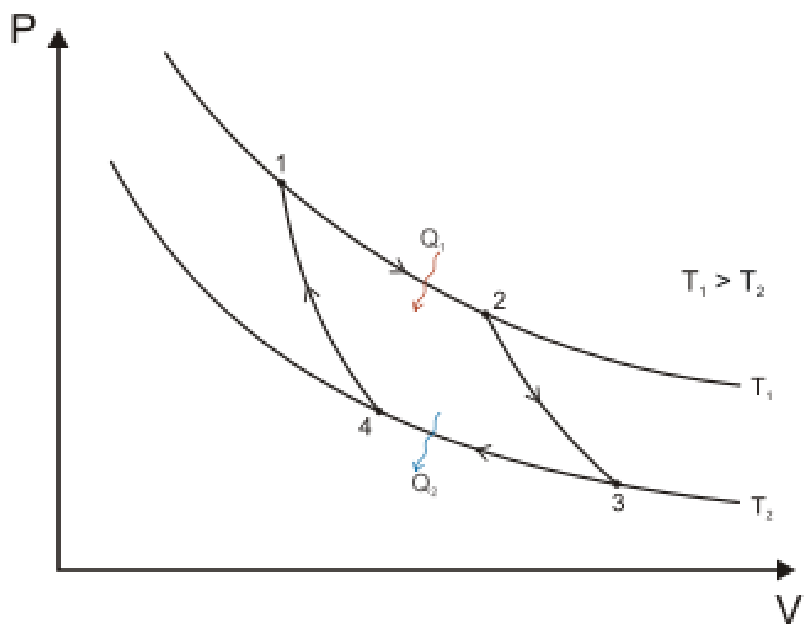


Figure 1: A Carnot cycle illustrated on a PV diagram to illustrate the work done.

3. **Reversible isothermal compression of the gas at the "cold" temperature, T_C. (isothermal heat rejection)** (3 to 4 on Figure 1, *C* to *D* on Figure 2) Now the gas is exposed to the cold temperature reservoir while the surroundings do work on the gas by compressing it (such as through the return compression of a piston), while causing an amount of heat energy Q_2 and of entropy $\Delta S_c = \frac{Q_c}{T_c}$ to flow out of the gas to the low temperature reservoir. (This is the same amount of entropy absorbed in step 1.) This work is less than the work performed on the surroundings in step 1 because it occurs at a lower pressure given the removal of heat to the cold reservoir as the compression occurs (i.e. the resistance to compression is lower under step 3 than the force of expansion under step 1).

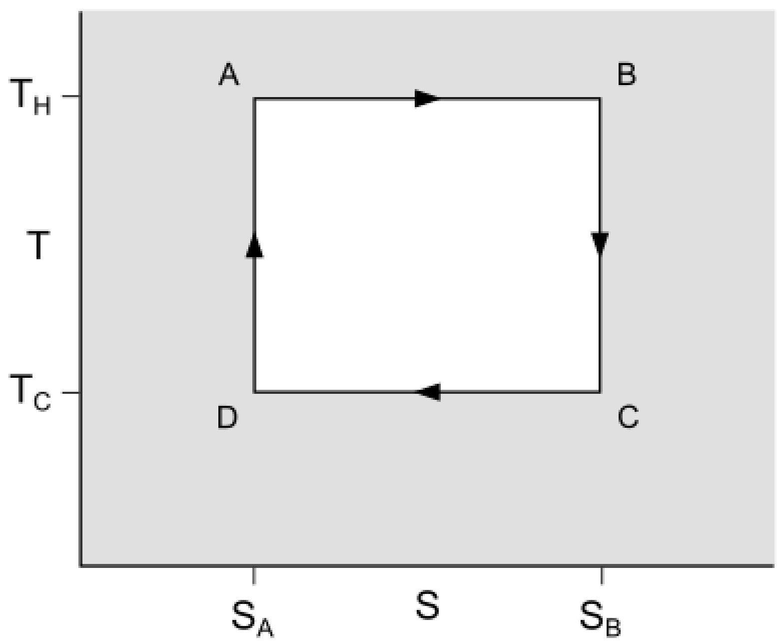


Figure 2: A Carnot cycle acting as a heat engine, illustrated on a temperature-entropy diagram.

4. **Isentropic compression of the gas (isentropic work input).** (4 to 1 on Figure 1, *D* to *A* on Figure 2) Once again the piston and cylinder are assumed to be thermally insulated and the cold temperature reservoir is removed. During this step, the surroundings continue to do work to further compress the gas and both the temperature and pressure rise now that

the heat sink has been removed. This additional work increases the internal energy of the gas, compressing it and causing the temperature to rise to T_H. The entropy remains unchanged. At this point the gas is in the same state as at the start of step 1.

Efficiency of a Real Heat Engine:

For a real heat engine, the total thermodynamic process is generally irreversible. The working fluid is brought back to its initial state after one cycle, and thus the change of entropy of the fluid system is 0, but the sum of the entropy changes in the hot and cold reservoir in this one cyclical process is greater than 0.

The internal energy of the fluid is also a state variable, so its total change in one cycle is 0. So the total work done by the system *W*, is equal to the heat put into the system Q_H minus the heat taken out Q_C.

$$W = Q_H - Q_C \qquad (1)$$

For real engines, sections 1 and 3 of the Carnot Cycle; in which heat is absorbed by the "working fluid" from the hot reservoir, and released by it to the cold reservoir, respectively; no longer remain ideally reversible, and there is a temperature differential between the temperature of the reservoir and the temperature of the fluid while heat exchange takes place.

During heat transfer from the hot reservoir at T_H to the fluid, the fluid would have a slightly lower temperature than T_H, and the process for the fluid may not necessarily remain isothermal. Let ΔS_H be the total entropy change of the fluid in the process of intake of heat.

$$\Delta S_H = \int_{Q_{in}} \frac{dQ_H}{T} \qquad (2)$$

Where, the temperature of the fluid *T* is always slightly lesser than T_H , in this process.

So, one would get,

$$\frac{Q_H}{T_H} = \frac{\int dQ_H}{T} \leq \Delta S_H \qquad (3)$$

Similarly, at the time of heat injection from the fluid to the cold reservoir one would have, for the magnitude of total entropy change ΔS_C of the fluid in the process of expelling heat:

$$\Delta S_C = \int_{Q_{out}} \frac{dQ_C}{T} \leq \frac{\int dQ_C}{T_C} \leq \frac{Q_c}{T_C} \qquad (4)$$

where, during this process of transfer of heat to the cold reservoir, the temperature of the fluid T is always slightly greater than T_C.

We have only considered the magnitude of the entropy change here. Since the total change of entropy of the fluid system for the cyclic process is 0, we must have

$$\Delta S_H = \Delta S_C \qquad (5)$$

The previous three equations combine to give:

$$\frac{Q_c}{T_C} \geq \frac{Q_H}{T_H} \qquad (6)$$

Equations (**1**) and (**6**) combine to give

$$\frac{W}{Q_H} \leq 1 - \frac{T_c}{T_H} \qquad (7)$$

Hence,

$$\eta \leq \eta_{max} \qquad (8)$$

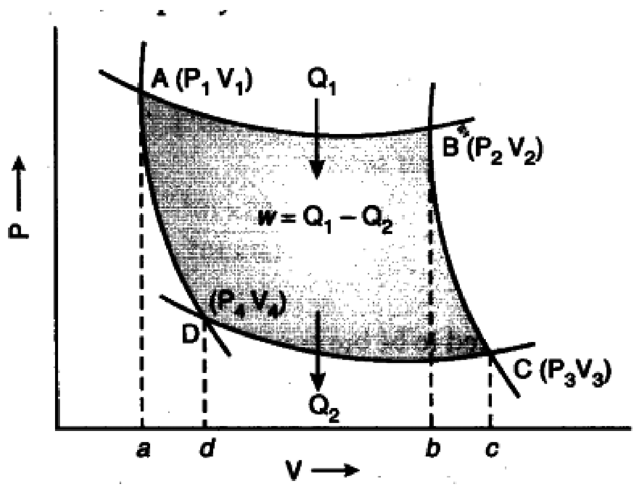
Where, $\eta = \frac{W}{Q_H}$ is the efficiency of the real engine, and η_{max} is the efficiency of the Carnot engine working between the same two reservoirs at the temperatures T_H and T_c . For the Carnot engine, the entire process is 'reversible', and Equation (**6**) is an equality.

Hence, the efficiency of the real engine is always less than the ideal Carnot engine.

Equation (***6***) signifies that the total entropy of the total system(the two reservoirs + fluid) increases for the real engine, because the entropy gain of the cold reservoir as Q_c flows into it at the fixed temperature T_c, is greater than the entropy loss of the hot reservoir as Q_H leaves it at its fixed temperature T_H. The inequality in Equation (**6**) is essentially the statement of the Clausius theorem.

Derive the equation to find the efficiency of Carnot engine cycle:

It is defined as ratio of net mechanical work done per cycle by the gas to the amount of heat energy absorbed per cycle from the source



$$\eta = \frac{W}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

In isothermal process

$$P_1V_1 = P_2V_2 \qquad ...(1)$$

As $B(V_2, P_2)$ and $C(V_3, P_3)$ lie on same adiabatic

$$P_2V_2^{\gamma} = P_3V_3^{\gamma} \qquad ...(2)$$

Again C and D lie on same isothermal

$$P_3V_3^{\gamma} = P_4V_4^{\gamma} \qquad ...(3)$$

Finally D and A lie on same adiabatic

$$P_4V_4^{\gamma} = P_1V_1^{\gamma} \qquad ..(4)$$

Multiplying (1), (2), (3), (4),

$V_2^{\gamma-1} + V_4^{\gamma-1} = V_1^{\gamma-1} + V_3^{\gamma-1}$

$$(V_2V_4) = (V_1V_3)$$

$$V_2V_4 = V_1V_3$$

$$\frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\log_e \frac{V_2}{V_1} = \log_e \frac{V_3}{V_4}$$

Dividing (4) by (2),

$$\frac{Q_2}{Q_1} = \frac{KT_2 \log_e V_3/V_4}{KT_1 \log_e V_2/V_1}$$

$$= \frac{T_2}{T_1}$$

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

This is the efficiency of a carnot heat engine.

Problem 1: Water near the surface of a tropical ocean has a temperature of 298.2K (250°C), whereas water 700m beneath the surface has a temperature of 280.2K (7.0°C). It has been proposed that the warm water be used as the hot reservoir and the cool water as the cold reservoir of a heat engine. Find the maximum possible efficiency for such an engine.

Solution:

Given data,

T_h= 298.2 K

T_l= 280.2 K

The maximum possible efficiency is the efficiency that a carnot engine would have operating between these two temperatures.

Substitute the value in the corresponding formula ,

$$\eta = \left(\frac{T_h - T_l}{T_h}\right)$$

$$\eta = \left(\frac{298.2 - 280.2}{298.2}\right)$$

η = 0.060 (6.0%)

Therefore, the maximum possible efficiency is 6 %.

Problem 2: The minimum power required to drive a heat pump which maintains a house at 20°C is 3KW. If the outside temperature is 3°C, estimate the amount of heat which the house loses per minute.

Solution:

For minimum power requirement the heat pump must operate on the reverse cycle.

$T_h = 293.15$
$T_l = 276.15$

$$\eta = \left(\frac{T_h - T_l}{T_h} \right)$$

$$\eta = \left(\frac{293.15 - 276.15}{293.15} \right)$$

$\eta = 0.0579$ (5.7%)

Therefore, the maximum possible efficiency is 5.7%.