Plank's Equation:

Probably the most straight forward expression available for computing freezing time was derived by Plank. The equation utilized for computation purposes can be derived for various geometries of product. The three basic equations utilized in the derivation account for heat transfer in various phases of the product during freezing.

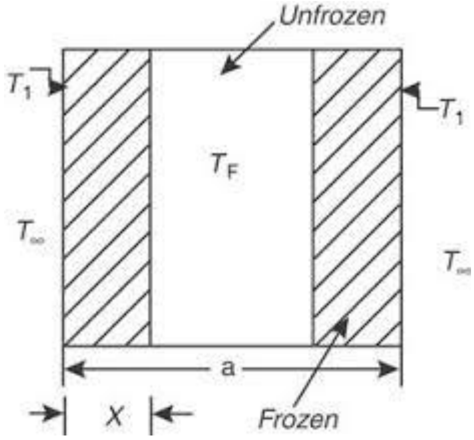

Fig: Schematic illustration of one-dimensional freezing of a product section used to derive Plank's equation

The first expression is the basic heat-conduction equation for the frozen product region which has a variable thickness of x as follows:

$$q = \frac{A(T_S - T_F)k}{x} \qquad (1)$$

Where T_F is the initial freezing point of the product and the temperature which exists in all unfrozen regions of the product, and k is the thermal conductivity of the frozen material. The second expression describes the heat transfer through the film which exists at the product surface, and may be expressed as:

$$q = h_c A(T_\infty - T_s) \qquad (2)$$

where h_c is a convective heat-transfer coefficient which may exist at the product surface. Equations (1) and (2) can be combined into one expression to account for heat transfer in series as follows:

$$q = \frac{A(T_F - T_\infty)}{\frac{1}{h_c} + \frac{x}{k}} \qquad (3)$$

which eliminates the need for knowledge of the surface temperature. The third equation, describing the rate at which is being generated at the freezing front, is as follows:

$$q = AL\rho \frac{dx}{dt} \qquad (4)$$

where the differential (dx/dt) represents the velocity of the freezing front. By equating equations (3) and (4) and by integration between the appropriate limits, the following expression for freezing time is obtained:

$$t_F = \frac{\rho L}{T_F - T_\infty}\left[\frac{a'}{2h_c} + \frac{a'^2}{8k}\right] \qquad (5)$$

where a' represents the total thickness of the slab being frozen. By introduction of the appropriate constants, the most general form of Plank's equation is obtained as follows:

$$t_F = \frac{\rho L}{T_F - T_\infty}\left[\frac{Pa'}{h_c} + \frac{Ra'^2}{k}\right] \qquad (6)$$

where P and R are constants which will vary depending on the geometry of the material being frozen. As is obvious, these constants are ½ and 1/8 for P and R, respectively, in the case of an infinite slab. For a sphere, P = 1/6 and R = 1/24, while for a infinite cylinder, P = ¼ and R = 1/16. The dimension,a', which is the thickness of the infinite slab, becomes the diameter of a cylinder and a sphere.

Problem-1: A spherical food product is being frozen in an air-blast wind tunnel. The initial product temperature is 10°C and the cold air -15°C. The product has a 7-cm diameter with density of 1,000 kg/m^3. The initial freezing temperature is -1.25°C, and the latent heat of fusion is 250 kJ/kg. Compute the freezing time.

Given: Initial product temperature, T_i = 10°C
Air temperature, T_∞ = -15°C
Initial freezing temperature, T_F = -1.25°C
Product diameter, a' = 7 cm (0.07 m)
Product density, ρ = 1000 kg/m3
Thermal conductivity of frozen product, k = 1.2 W/m.k
Latent heat, L = 250 kJ/kg
Shape constants for spheres: P = 1/6, R = 1/24

Convective heat-transfer coefficient, $h_c = 50\ W/m^2.k$

Solution: Freezing time calculation:

We Know that,

$$t_F = \frac{\rho L}{T_F - T_\infty}\left[\frac{Pa'}{h_c} + \frac{Ra'^2}{k}\right]$$

$$t_F = \frac{1000 kg/m^3 \times 250 kJ/kg}{[-1.25\ ^oC - (-15\ ^oC)]}\left[\frac{0.07 m}{6\times(50\ W/m^2.K)} + \frac{(0.07 m)^2}{24\times(1.2\ W/m.K)}\right]$$

$$= 18182\ \frac{kJ}{m^3.^oC}\times\left[2.33\times10^{-4}\frac{m^3.K}{W} + 1.7\times10^{-4}\frac{m^3.K}{W}\right]$$

$$= 7.33\ \ kJ/W$$

$$Since\ \ 1KJ = 1000J\ \ and\ \ 1W = 1\ \ J/s$$

$$t_F = \frac{7.33\times1000\ \ J}{1\ J/s} = 7.33\times10^3\ s = 2.04\ \ hr$$

Refrigeration: *Refrigeration* is defined as the process of extracting heat from a lower-temperature heat source, substance, or cooling medium and transferring it to a higher-temperature heat sink. Refrigeration maintains the temperature of the heat source below that of its surroundings while transferring the extracted heat, and any required energy input, to a heat sink, atmospheric air, or surface water.

Refrigeration System: A refrigeration system is a combination of components and equipment connected in a sequential order to produce the refrigeration effect. The refrigeration systems commonly used for air conditioning can be classified by the type of input energy and the refrigeration process as follows:

1. ***Vapor compression systems***. In vapor compression systems, compressors activate the refrigerant by compressing it to a higher pressure and higher temperature level after it has produced its refrigeration effect. The compressed refrigerant transfers its heat to the sink and is condensed to liquid form. This liquid refrigerant is then throttled to a low-pressure, low temperature vapor to produce refrigerating effect during evaporation. Vapor compression systems are the most widely adopted refrigeration systems in both comfort and process air conditioning.

2. ***Absorption systems***. In an absorption system, the refrigeration effect is produced by thermal energy input. After absorbing heat from the cooling medium during evaporation, the vapor refrigerant is absorbed by an absorbent medium. This solution is then heated by direct-fired furnace, waste heat, hot water, or steam. The refrigerant is again vaporized and then condensed to liquid to begin the refrigeration cycle again.

3. ***Air or gas expansion systems***. In an air or gas expansion system, air or gas is compressed to a high pressure by mechanical energy. It is then cooled and expanded to a low pressure. Because the temperature of air or gas drops during expansion, a refrigeration effect is produced.

Some Definitions:

Refrigerants: A refrigerant is the primary working fluid used for absorbing and transmitting heat in a refrigeration system. Refrigerants absorb heat at a low temperature and low pressure and release heat at a higher temperature and pressure. Most refrigerants undergo phase changes during heat absorption—evaporation—and heat releasing—condensation.

Example: Ammonia, Pentane, Carbon tetra chloride, Trichlorofluromethane etc.

Cooling Media: A cooling medium is the working fluid cooled by the refrigerant to transport the cooling effect between a central plant and remote cooling units and terminals. In a large, centralized system, it is often more economical to use a coolant medium that can be pumped to remote locations where cooling is required. Chilled water, brine, and glycol are used as cooling media in many refrigeration systems. The cooling medium is often called a secondary refrigerant, because it obviates extensive circulation of the primary refrigerant.

Liquid Absorbents: A solution known as liquid absorbent is often used to absorb the vaporized refrigerant (water vapor) after its evaporation in an absorption refrigeration system. This solution, containing the absorbed vapor, is then heated at high pressure. The refrigerant vaporizes, and the solution is restored to its original concentration for reuse. Lithium bromide and ammonia, both in a water solution, are the liquid absorbents used most often in absorption refrigerating systems.

Vapor Compression Refrigeration Cycle:

A vapour compression refrigeration system is an improved type of air refrigeration system in which a suitable working substance, termed as refrigerant is used. It condensed and evaporates at temperatures and pressures close to the atmospheric conditions. The refrigerants usually used for this purpose are ammonia, carbon dioxide and sulphur dioxide.

Advantage and disadvantages of vapour compression and air refrigeration system:

Advantage:

1. It has smaller size for given capacity of refrigeration.
2. It has less running cost.
3. It can be employed over a large range of temperatures

4. The coefficient of performance is quite high.

Disadvantages:

1. The initial cost is high
2. The prevention of leakage of refrigerant is the major problem in vapour compression system.

The refrigeration cycle is shown in Figure below and can be broken down into the following stages:

1 – 2 Low-pressure liquid refrigerant

In the evaporator absorbs heat from its surroundings, usually air, water or some other process liquid. During this process it changes its state from a liquid to a gas, and at the evaporator exit is slightly superheated.

2 – 3 The superheated vapour

Enters the compressor where its pressure is raised. The temperature will also increase, because a proportion of the energy put into the compression process is transferred to the refrigerant.

3 – 4 The high pressure superheated gas

Passes from the compressor into the condenser. The initial part of the cooling process (3-3a) superheats the gas before it is then turned back into liquid (3a-3b). The cooling for this process is usually achieved by using air or water. A further reduction in temperature happens in the pipe work and liquid receiver (3b - 4), so that the refrigerant liquid is sub-cooled as it enters the expansion device.

4 - 1 The high-pressure sub-cooled liquid

Passes through the expansion device, which both reduces its pressure and controls the flow into the evaporator.

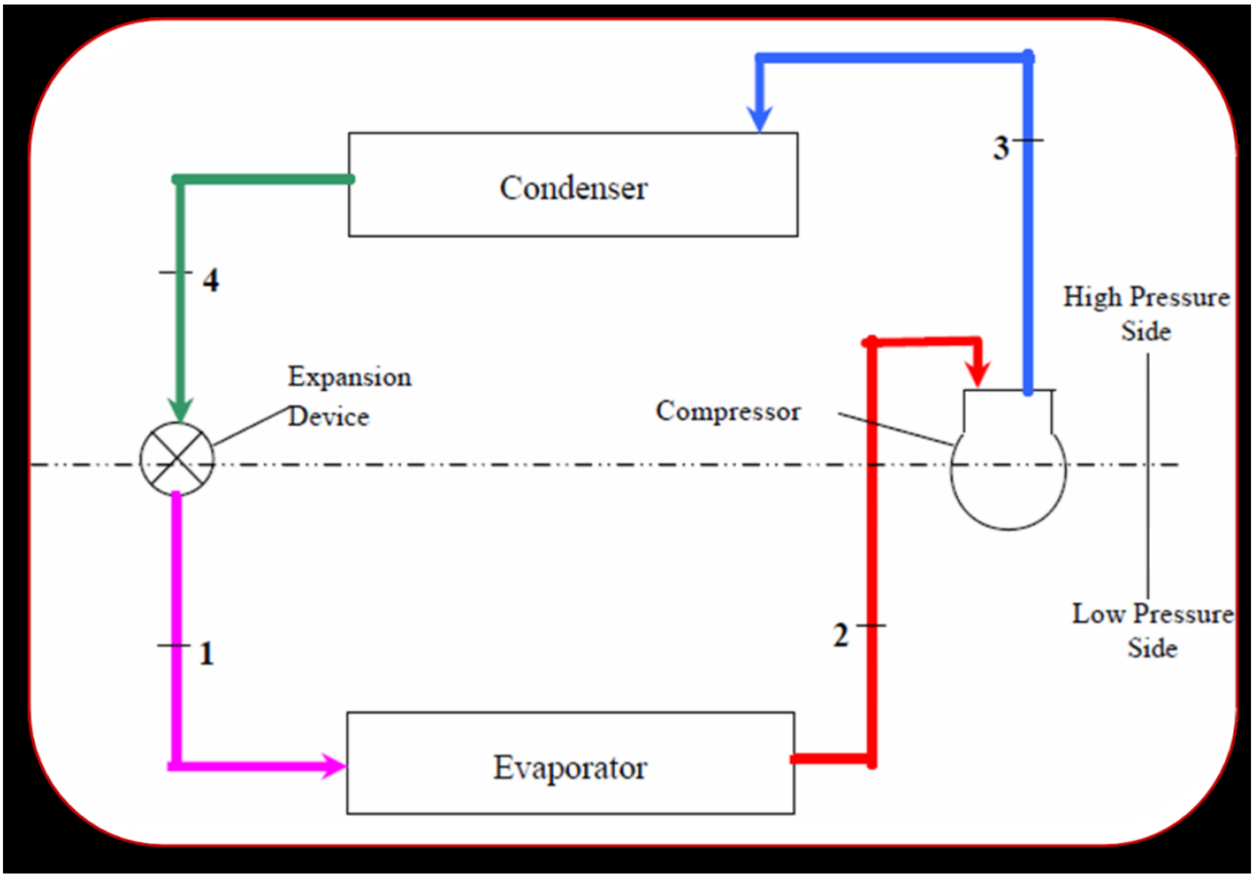

Figure 1. Schematic representation of the vapour compression refrigeration cycle